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MA 425/525
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Homework 2

Due September 4th on paper at the beginning of class. Justify your answers. Please let me know if you have a question or find a mistake. The book is `https://archive.org/details/complex-variables-2ed-dover-1999-fisher/page/n23/mode/2up`.

Do 1.1.15 from page 9 and 1.2.1 (include a sketch), 1.2.2 (include a sketch), 1.2.15 (include a sketch), 1.2.22, 1.2.24 (include a sketch and also write all solutions in the form $z = a + bi$ with a and b real) and 1.2.30 from pages 20 – 21.

There are some hints on the next page.

Hints:

- For 1.1.15, show that $|z-w|^2 = |z|^2 - 2\,\mathrm{Re}(z\bar{w}) + |w|^2$ and use the definition of equilateral triangle. Note that there is an outline of a solution on page 397 but you should be sure to fill in all the missing steps carefully if you use it. For this one, and the other 'if and only if' problems, make sure you have separate parts of your solution for the 'if' and 'only if' parts of the problem.
- For 1.2.15, recall that you can find the center of the circle passing through given points A, B, and C by finding the intersection point of the perpendicular bisectors of the segments AB, AC, and BC. In this example the perpendicular bisectors of the given points are nice.
- For 1.2.22, to show that $|z+w| = |z| + |w|$ implies $z = sw$ for some $s > 0$, square both sides and use the formula for $\cos\alpha$ from the middle of page 8.
- For 1.2.30, use the formula for $|z-w|^2$ from problem 1.1.15 to get a formula for $|z^2| + \mathrm{Re}(Az)$ in terms of $|z-w|^2$ and $|w|^2$ for a suitable w.